

# The information content of implied volatility in light of the jump/continuous decomposition of realized volatility

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## ABSTRACT

In the framework of encompassing regressions, we assess the information content of the jump/continuous components of historical volatility when implied volatility is included as an additional regressor. Our empirical application focuses on daily and intradaily data for the S&P100 and S&P500 indexes, and daily data for the associated VXO and VIX implied volatility indexes. Our results show that the total explanatory power of the encompassing regressions barely changes when the jump/continuous components are included, although the weekly and monthly continuous components are usually significant. In contrast, the jump components of realized volatility are much less relevant, although the monthly jump component sometimes does convey information. This evi-

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dence supports the view that implied volatility has a very high information content, even when extended decompositions of past realized volatility are used.

# 1. Introduction

The information content of implied volatility has been well documented in many empirical studies. Most papers look at whether implied volatility helps predict future volatility by estimating either encompassing regressions or ARCH-type models. In both settings, volatility forecasts are initially based on historical volatility. When implied volatility is included, the important research issues focus on whether it supplies additional information and whether the historical volatility is still relevant, i.e. what is the additional information content of historical volatility. As the focus of these studies is on implied volatility, the historical volatility that is used is often a rather crude measure (lagged realized volatility in an encompassing regression, lagged squared error term in the ARCH framework). In contrast, we suggest that more sophisticated measures of historical volatility should be taken into account when assessing the information content of implied volatility. In this paper, we use the recent continuous/jump decomposition of historical realized volatility suggested by Andersen, Bollerslev, and Diebold (2003). This decomposition also paves the way for a ‘time structure’ (daily, weekly, monthly decomposition) for each volatility component. The latter decompositions are motivated by the recent Heterogeneous Autoregressive model of the Realized Volatility (HAR-RV) introduced by Corsi (2003). Hence, we look at the information content of the continuous and jump components of historical volatility in encompassing regressions which also feature implied volatility as an additional regressor. Our contribution to the literature is thus to provide a more sophisticated and promising econometric framework to shed light on the information content of implied volatility when the latter is competing against *extended* measures of historical volatility computed by market participants.

The empirical application focuses on the S&P100 and S&P500 indexes and associated VXO and VIX implied volatility indexes. Our results show that the weekly and monthly continuous components of historical volatility convey information beyond the implied volatility forecasts. In other words, when the weekly/monthly components are included in the encompassing regressions there is a shift in explanatory power away from the implied volatility

measure towards the extended historical measures. However, the total explanatory power of the regression barely increases (the modified  $R^2$  suggested by Andersen, Bollerslev, and Meddahi, 2002, hovers around 0.7). This suggests that the implied volatility does indeed subsume most relevant volatility information and also indicates that the extended decomposition of the past historical volatility does not really help deliver better volatility forecasts. Jump components provide much less information, although the monthly jump component of historical volatility is sometimes significantly negative. This could suggest that agents tend to overreact to sustained periods of ‘jump volatility’.

The rest of the paper is structured as follows. After this introduction, we characterize the implied volatility and the continuous/jump components of realized volatility in Section 2. Section 3 presents the econometric framework and the finance interpretation of the suggested encompassing regressions. Section 4 details the datasets and Section 5 provides the empirical application. Finally, Section 6 concludes.

## **2. Implied and realized volatility**

### **2.1. Implied volatility**

The information content of implied volatility has been examined in many studies for the last 20 years. Most empirical studies assess the information content of implied volatility by either estimating so-called encompassing regressions (e.g. Christensen and Prabhala, 1998; Corrado and Miller, 2004), or by estimating ARCH-type models with the lagged implied volatility included as an additional variable in the conditional variance specification (e.g. Day and Lewis, 1992; Blair, Poon, and Taylor, 2001), or by looking at the added value of the implied volatility in a finance application (e.g. Giot, 2005a, 2005b). Our contribution relies on encompassing regressions that take into account the continuous/jump decomposition of the so-called realized volatility introduced by Andersen and Bollerslev (1998b). Without this decomposition, the en-

compassing regression usually regress the  $h$ -day observed realized volatility against the  $h$ -day implied volatility forecast and lagged values of the dependent variable (this is the approach commonly used in the literature). The extended regressions that feature the continuous and jump components of realized volatility are described in the next section.

While some authors use their own computation of implied volatility, others rely on the availability of implied volatility indexes to derive  $h$ -day-ahead implied volatility forecasts. This is also the chosen framework for the empirical application of the paper. In these latter studies, the square root of time rule is used to switch from the ‘natural’ horizon of the implied volatility index to the required  $h$ -day horizon. Our study focuses on the S&P100 and S&P500 indexes, for which the corresponding implied volatility indexes (VXO and VIX indexes) are computed by the CBOE. The construction of the VXO index is described in Whaley (1993) or Fleming and Whaley (1995). The new VIX index is detailed in the CBOE technical document “The New CBOE Volatility Index - VIX” available from the CBOE website, or in Giot (2005a). While we refer the reader to these papers for a detailed presentation of the construction of the implied volatility indexes, it is important to stress that the VXO and VIX indexes are computed differently. Indeed, the VXO is based on the ‘old VIX’ formula, i.e. it is an implied volatility backed out from 8 at-the-money options on the S&P100 index. In contrast, the VIX index is computed from options on the S&P500 index and the options taken into account feature a wide strike price variation. Consequently, the VIX should better integrate the volatility information contained in the option prices, i.e. not only at-the-money options but also options for which the strike price can be far away from the price of the index. For both implied volatility indexes, their natural  $h$ -day-ahead horizon is equal to 22 trading days (by construction), i.e. they are meant to deliver 22-day-ahead volatility forecasts for the underlying stock index. As mentioned above, the square root of time rule is then used to switch to another time horizon: given  $VXO_t$  or  $VIX_t$  available at the end of day  $t$  and expressed in annualized terms, the  $h$ -day-ahead implied volatility forecast is equal to  $\sqrt{\frac{h}{365}} VXO_t$  for the S&P100 index, while it is  $\sqrt{\frac{h}{365}} VIX_t$  for the S&P500 index.

## 2.2. Realized volatility and its two components

The recent widespread availability of databases providing the intraday prices of financial assets (stocks, stock indexes, bonds, currencies,...) has led to new developments in applied econometrics and quantitative finance. The notion of realized volatility has been introduced in the literature by Andersen and Bollerslev (1998b). The most interesting feature of realized volatility is that it provides, under certain conditions, a consistent nonparametric estimate of the asset price variability.

Let us consider the following continuous-time jump diffusion process for the logarithmic price of an asset  $p(t)$ :

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t), t \in [0, T], \quad (1)$$

where  $\mu(t)$  is a continuous locally bounded variation process, the volatility process  $\sigma(t)$  is strictly positive,  $W(t)$  denotes a standard Brownian motion,  $dq(t)$  is a counting process with  $dq(t) = 1$  corresponding to a jump at time  $t$  and  $dq(t) = 0$  otherwise and  $\kappa(t)$  refers to the size of the corresponding jumps.

The return over the discrete interval  $[t - \Delta, t]$ ,  $\Delta > 0$ , is then given by  $r(t, \Delta) \equiv p(t) - p(t - \Delta)$ . Following Andersen and Bollerslev (1998b), the daily realized volatility is computed by summing the corresponding  $1/\Delta$  high-frequency intraday squared returns<sup>1</sup>,  $RV_{t+1}(\Delta) \equiv \sum_{j=1}^{1/\Delta} r_{t+j\cdot\Delta, \Delta}^2$ . As emphasized by many authors, see Barndorff-Nielsen and Shephard (2002) among others,

$$RV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(s)ds + \sum_{t < s \leq t+1} \kappa^2(s), \quad (2)$$

as  $\Delta \rightarrow 0$ . Equation (2) then implies that the realized volatility is a consistent estimate of the integrated volatility in the absence of jumps.

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<sup>1</sup>Without loss of generality  $1/\Delta$  is assumed to be an integer.

In an important contribution to the realized volatility literature, Barndorff-Nielsen and Shephard (2003) show that, unlike the realized volatility, the so-called bi-power variation measures constructed from the summation of appropriately scaled cross-products of adjacent high-frequency absolute returns gives a consistent estimate of the integrated volatility in the presence of jumps. More formally, following Barndorff-Nielsen and Shephard (2003, 2004, 2005), the standardized realized bi-power variation measure is defined as:

$$BV_{t+1}(\Delta) \equiv \mu_1^{-1} \sum_{j=2}^{1/\Delta} |r_{t+j\cdot\Delta,\Delta}| |r_{t+(j-1)\cdot\Delta,\Delta}|, \quad (3)$$

where  $\mu_1 \equiv \sqrt{2/\pi}$ . It follows that

$$BV_{t+1}(\Delta) \rightarrow \int_t^{t+1} \sigma^2(s) ds, \quad (4)$$

as  $\Delta \rightarrow 0$ . Consequently, the jump component is consistently measured by:

$$J_{t+1}(\Delta) \equiv \max[RV_{t+1}(\Delta) - BV_{t+1}(\Delta), 0]. \quad (5)$$

In this framework, Andersen, Bollerslev, and Diebold (2003) propose an elegant and simple statistical procedure for testing the significance of the jumps. Their procedure requires the computation of the standardized realized tri-power quarticity (a consistent estimate of the integrated quarticity  $\int_t^{t+1} \sigma^2(s) ds$ ):

$$TQ_{t+1}(\Delta) \equiv \Delta^{-1} \mu_{4/3}^{-3} \sum_{j=3}^{1/\Delta} |r_{t+j\cdot\Delta,\Delta}|^{4/3} |r_{t+(j-1)\cdot\Delta,\Delta}|^{4/3} |r_{t+(j-2)\cdot\Delta,\Delta}|^{4/3}, \quad (6)$$

where  $\mu_{4/3} \equiv 2^{2/3} \Gamma(7/6) \Gamma(1/2)^{-1}$ . Accordingly, a jump is said to be significant when

$$Z_{t+1}(\Delta) \equiv \frac{\log(RV_{t+1}(\Delta)) - \log(BV_{t+1}(\Delta))}{\sqrt{\Delta(\mu_1^{-4} + 2\mu_1^{-2} - 5)TQ_{t+1}(\Delta)BV_{t+1}(\Delta)^{-2}}} > \Phi_\alpha, \quad (7)$$

where  $\Phi_\alpha$  is the  $\alpha$  quantile of the standard normal distribution.<sup>2</sup>

### 3. Econometric framework

#### 3.1. Notation and econometric models

In this section, we build on the previous characterization of implied volatility and realized volatility to define the variables we are working with and introduce the encompassing regressions. Let  $t$  be the index for the daily observations and  $h$  be the forward-looking horizon (expressed in number of days). On a daily basis, the following variables are defined.  $RV_t^1$  is the daily realized volatility over the  $[t : t + 1]$  time horizon computed from the 20-minute intradaily returns observed over the same period of time (we full characterize our datasets in the next section). Like in Andersen, Bollerslev, Diebold, and Ebens (2001), the daily realized volatility is computed as the sum of the squared intraday returns. Following Andersen, Bollerslev, and Diebold (2003) and the discussion presented in Section 2.2,  $RV_t^1$  is split into  $RC_t^1$  and  $RJ_t^1$ .  $RV_t^h$  is the realized volatility over  $[t : t + h]$ , computed by summing the  $RV_t^i$ , for  $i = 1 \dots h$ .  $IMP_t^h$  is the implied volatility observed at the end of day  $t$  and scaled such that it forecasts the next h-day volatility:  $IMP_t^h = \sqrt{\frac{h}{365}} IMP_t^1$ , with  $IMP_t^1 = VIX_t$  or  $IMP_t^1 = VXO_t$  depending on the chosen underlying stock index.<sup>3</sup> Given the decomposition of  $RV_t^1$  into  $RC_t^1$  and  $RJ_t^1$ , we define  $RC_t^d = RC_t^1$ ,  $RJ_t^d = RJ_t^1$ ,  $RC_t^w = (RC_t^1 + \dots + RC_{t-4}^1)/5$ ,  $RJ_t^w = (RJ_t^1 + \dots + RJ_{t-4}^1)/5$ ,  $RC_t^m = (RC_t^1 + \dots + RC_{t-21}^1)/22$  and  $RJ_t^m = (RJ_t^1 + \dots + RJ_{t-21}^1)/22$ . These latter variables are easily interpreted as the daily, weekly and monthly continuous and jump components (of the realized volatility) respectively. For example,  $RC_t^w$  is the continuous component of the weekly

<sup>2</sup>To ensure that the measurements of the continuous sample path variation and the jump component add up to the realized volatility, non-significant jumps are transferred to the continuous sample path variation  $BV_{t+1}(\Delta)$ . See Andersen, Bollerslev, and Diebold (2003) for more details.

<sup>3</sup>Note that the most simple assessment of the implied volatility forecast would thus be a regression of  $RV_t^h$  against  $IMP_t^h$ .

realized volatility ending on day  $t$ ;  $RJ_t^m$  is the jump component of the monthly realized volatility ending on day  $t$ .

By definition,  $IMP_t^h$  and  $RV_t^h$  are based on overlapping data when as  $h > 1$ . To estimate the extended encompassing regressions, we need to define related variables in a non-overlapping framework (this issue was first highlighted by Christensen and Prabhala, 1998). Let  $k$  be the index for the non-overlapping observations ( $1 \dots K$ ). The following variables are then defined.  $\{RV_k^h\}$  is the subset of  $\{RV_t^h\}$  such that 1 out of  $h$  observations is retrieved.  $IMP_k^h$ ,  $RC_k^d$ ,  $RC_k^w$ ,  $RC_k^m$ ,  $RJ_k^d$ ,  $RJ_k^w$  and  $RJ_k^m$  are defined similarly. We stress that all variables are interpreted exactly as above. Since  $RC_t^m$  and  $RJ_t^m$  are based on an aggregation of the last 22 days,  $RC_k^m$  and  $RJ_k^m$  are the only variables that are still based on overlapping data when  $h = 5$  or  $h = 10$  days (hence we shall use the Newey-West procedure when estimating the regressions). This is however consistent with the procedures put forth by Corsi (2003). Finally, all variables are transformed in logs, yielding  $lRV_k^h = \ln(RV_k^h)$ ,  $lIMP_k^h = \ln(IMP_k^h)$ , ... The use of the log realized volatility (and other log variables) in the regressions below is recommended to ensure that the probability density of the error term is close to the normal density (Giot and Laurent, 2004).

To assess the information content of implied volatility in the extended framework that features the continuous/jump decomposition, we estimate the following models:

### Model 1

$$lRV_k^h = \beta_0 + \beta_1 lIMP_k^h + \varepsilon_k; \quad (8)$$

### Model 2

$$lRV_k^h = \beta_0 + \beta_1 lIMP_k^h + \delta_d lRC_k^d + \delta_w lRC_k^w + \delta_m lRC_k^m + \gamma_d lRJ_k^d + \gamma_w lRJ_k^w + \gamma_m lRJ_k^m + \varepsilon_k; \quad (9)$$

### Model 3

$$lRV_k^h = \beta_0 + \beta_1 lIMP_k^h + \gamma_d lRJ_k^d + \gamma_w lRJ_k^w + \gamma_m lRJ_k^m + \varepsilon_k. \quad (10)$$

Next to these specifications, we also assess whether these regressions exhibit heteroscedasticity. We consider F tests where we include the jump/continuous components as explanatory variables. Regarding model 3 for example, the heteroscedasticity specification is:

$$e_k^2 = \alpha_0 + \alpha_1 IIMP_k^h + \alpha_2 IRJ_k^d + \alpha_3 IRJ_k^w + \alpha_4 IRJ_k^m + v_k \quad (11)$$

with  $H_0: \alpha_1 = 0$  and  $\alpha_2 = 0$  and  $\alpha_3 = 0$  and  $\alpha_4 = 0$ .

### 3.2. Interpretation of the models

The interpretation of the regressions is as follows. Model 1 is the most simple forecasting regression: the observed realized volatility is compared with the forecast derived from the implied volatility. This type of regression and the hypothesis tests that follow (e.g. is  $H_0: \beta_0 = 0$  and  $\beta_1 = 1$  not rejected?) have been well documented in the literature. Model 2 is the extended encompassing regression that features the daily, weekly and monthly continuous/jump components of the realized volatility. In contrast to the standard regressions previously estimated in the literature (e.g.  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \beta_2 IRV_{k-1}^h + \varepsilon_k$ ), we therefore assess the importance of the decomposition and ‘time-structure’ of historical volatility. For example, most studies based on  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \beta_2 IRV_{k-1}^h + \varepsilon_k$  usually conclude that  $\beta_2$  is not significant. This specification is however very restrictive as it implies that agents only take into account the lagged  $h$ -day realized volatility when forecasting the next  $h$ -day volatility in addition of the implied volatility. The continuous/jump decomposition also allows us to assess the role played by highly volatile markets regarding the quality of the volatility forecasts. Indeed, it is well known that economic agents rely on the continuous component of realized volatility to forecast future volatility (see Andersen, Bollerslev, and Diebold, 2002). But what is the impact of prolonged periods of jump volatility? It could be argued that agents tend to overreact and wrongly extrapolate this heightened volatility into the future. Last, does implied volatility subsume all relevant information? If this is true, the continuous and jump components

should not be significant. Model 3 is a restrictive version of model 2 where the continuous components are not included.

The heteroscedasticity tests can also be interpreted in a finance framework since it can be conjectured that a sustained period of jump volatility will lead to a much larger *uncertainty* in the forecasting performances of the encompassing regressions. In the same vein, Christensen and Prabhala (1998) and more recently Corrado and Miller (2005) study the impact of the ‘errors-in-variables’ problem in the encompassing regressions. Corrado and Miller (2005) suggest (although they do not call it the jump component of volatility) that volatility can also be decomposed into a ‘fundamental’ volatility and a ‘random’ volatility. The latter leads to the errors-in-variables problem. Our continuous/jump framework therefore also allows us to test if the precision of the regressions is affected by one (or both) of the volatility components.

## 4. The datasets

For the underlying stock indexes, the intraday data is provided by Tick Data (S&P100 and S&P500 cash indexes sampled at the 20-minute frequency; the data is linearly interpolated as in Muller, Dacorogna, Olsen, Pictet, Schwarz, and Morgeneegg, 1990, and Dacorogna, Muller, Nagler, Olsen, and Pictet, 1993). Regarding the VXO and VIX implied volatility indexes, the daily data is supplied by the CBOE. The time period is 1/2/1990 - 3/5/2003 for all indexes. Log returns are then defined and the daily realized volatility  $RV_t^1$  is computed as in Andersen and Bollerslev (1998a). Next, we decompose  $RV_t^1$  into its continuous and jump components,  $RC_t^1$  and  $RJ_t^1$ , and compute the daily, weekly and monthly continuous and jump components. In agreement with recent empirical papers on this topic, the jump threshold,  $\alpha$  in Equation 7, is set equal to 0.9999 (note that we assess the dependence of our results on the  $\alpha$  threshold later in the text). Finally, we switch to non-overlapping data with  $h$  successively equal to 5, 10 and 22 trading days. At the end of this process, we thus have 6 datasets, each dataset featuring the  $h$ -day realized volatility, 6 historical volatility variables and 1 implied volatility

forecast. To illustrate the information content of implied volatility at the 10-day and 22-day time horizons, we successively plot  $IMP_k^{10}$  vs  $RV_k^{10}$  and  $IMP_k^{22}$  vs  $RV_k^{22}$  for the two indexes in Figures 1 to 4. As expected from the literature on implied volatility indexes, the fit between the two series is very good, which bodes well for the assessment of the information content of implied volatility in Model 1.

## 5. Empirical results

Empirical results for models 1 (M1 in the table), 2 (M2) and 3 (M3) are reported in Tables 1, 2 and 3 for  $h$  equal to 5, 10 and 22 days respectively. In each table, the left (right) panel is for the S&P100 (S&P500) index. All regressions were estimated using the Newey-West option, yielding autocorrelation consistent results. In each panel, below the estimated coefficients, we also report the regressions  $R^2$  and adjusted  $R^2$  computed according to Andersen, Bollerslev, and Meddahi (2002). Indeed, Andersen, Bollerslev, and Meddahi (2002) show that the usual  $R^2$ s should be modified when assessing volatility forecasts in a realized volatility framework. They derive adjusted  $R^2$  measures that take the highlighted biases into account. We also give the P-values for the heteroscedasticity test described in Section 3.1.

The empirical results are quite similar for the six datasets. Results for the most simple model (M1) indicate that the  $\beta_1$  coefficient for the log implied volatility is almost equal to 1. For five cases out of six, we do not reject this hypothesis at the conventional level of significance. Because the constant ( $\beta_0$ ) is however different from 0, we reject the null hypothesis of unbiasedness (this is confirmed by a joint test on both coefficients). Nevertheless, the fact that  $\beta_1$  is almost equal to 1 supports the hypothesis that implied volatility almost moves one-to-one with future realized volatility. The large  $R^2$  and adjusted  $R^2$  values also indicate that implied volatility has a high information content: for the three horizons, the explicative power of implied volatility is close to 70%. Note that this number is remarkably stable, whatever the time horizon ( $h$ ) taken into account.

As far as model 2 is concerned, the inclusion of the continuous/jump components leads to a sharp decrease in the  $\beta_1$  coefficient; correspondingly, the weekly and monthly continuous components are significantly different from zero in almost all cases. In contrast, the jump components are much less significant. Although the weekly and monthly continuous components are significant in most cases, it is important to note that the  $R^2$  and adjusted  $R^2$  barely increase when switching from model 1 to model 2. Indeed, for both indexes and three horizons, some explicative power is transferred from the implied volatility to the continuous components but the total explicative power of the encompassing regression hardly increases. These facts tend to support the hypothesis that implied volatility has, by itself, a very high information content and that past 'extended' measures of realized volatility do not really bring valuable additional information.

Note that the coefficient for the monthly jump component is in some cases significantly negative and takes a rather large negative value. We interpret this as evidence that sustained periods of jump volatility lead to an overestimation of future realized volatility (either by the implied volatility index and/or by the continuous component of historical volatility). This is most likely due to agents overreacting to recent (but sustained) market conditions. Finally, model 3 sheds some light on the behavior of the implied volatility index when used in conjunction with the jump components of historical volatility. In that latter case, the empirical results (estimated coefficients for  $\beta_0$  and  $\beta_1$ ,  $R^2$  and adjusted  $R^2$ ) are very similar to those of Model 1. This supports the view that jump components do not bring forth valuable information (as far as forecasting is concerned).

Quite surprisingly, the heteroscedasticity tests (P-values are given at the bottom of each panel in the tables) show that the explanatory variables do not have any impact on the squared error terms for all specifications but one. Indeed, the null hypothesis of no heteroscedasticity is (almost) never rejected at the conventional 5% level. In light of the discussion at the end of Section 3.2, the precision of the encompassing regressions does not seem to be affected by the jump and continuous components of historical volatility. This result tends to support

the conclusions of Corrado and Miller (2005), who show that the errors-in-variables problem hardly affects the encompassing regressions (although they do conclude that forecast errors occurred much more frequently before 1994).

**Additional estimation results** To assess the robustness of our results, we implemented a series of changes regarding the definition of our variables and specification of the regressions. First, we looked at the impact of the jump component selection parameter. If this threshold is decreased, fewer intraday movements are classified as jumps. Second, we estimated models 1, 2 and 3 using a slightly different HAR specification. Indeed, in the current HAR specification of Corsi (2003),  $RC_t^w$  includes 1/5 of  $RC_t^d$ ,  $RC_t^m$  includes 1/22 of  $RC_t^d$ ,... We introduced a modified HAR specification such that  $RC_t^w = (RC_{t-1}^1 + \dots + RC_{t-4}^1)/4$ ,  $RC_t^m = (RC_{t-5}^1 + \dots + RC_{t-21}^1)/17$ ,... In this modified setting, variables  $RC_t^d$ ,  $RC_t^w$ ,  $RC_t^m$ ,  $RJ_t^d$ ,  $RJ_t^w$  and  $RJ_t^m$  no longer share volatility information. Third, we implemented rolling regressions for all models (fixed 3-year window length). The new empirical results were in line with those documented above and allowed us to conclude similarly.

## 6. Conclusion

This paper looks at the information content of the jump and continuous components of historical volatility in encompassing regressions that feature implied volatility as a regressor (Christensen and Prabhala, 1998, framework). In contrast to previous empirical studies, we therefore assess whether the continuous/jump decomposition of historical volatility and its time structure impact the explanatory power and information content of implied volatility.

Our empirical results for the S&P100 and S&P500 indexes show that the highlighted weekly and monthly continuous components convey information beyond implied volatility. In a few cases, the jump components are also significant. However, the total explanatory power of the encompassing regressions barely changes when switching from the most simple

model (only implied volatility is included) to the most sophisticated model (implied volatility and the full time structure decomposition of the continuous and jump components). This tends to support the fact that implied volatility does indeed subsume most relevant volatility information, even when ‘extended’ decomposition of the historical volatility are used. Finally, the precision of the regressions do not seem to be affected by either the continuous or jump components of volatility.

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**Table 1**  
**Encompassing regressions: models 1, 2 and 3:  $h = 5$  days**

	S&P100 index			S&P500 index		
	M1	M2	M3	M1	M2	M3
$\beta_0$	-0.47 (0.07)	0.43 (0.15)	-0.46 (0.07)	-0.72 (0.08)	0.66 (0.13)	-0.73 (0.08)
$\beta_1$	1.02 (0.04)	0.59 (0.07)	1.02 (0.05)	1.13 (0.06)	0.50 (0.06)	1.15 (0.06)
$\delta_d$		0.06 (0.04)			0.09 (0.04)	
$\delta_w$		0.25 (0.07)			0.25 (0.07)	
$\delta_m$		0.10 (0.08)			0.22 (0.07)	
$\gamma_d$		0.20 (0.09)	0.19 (0.09)		0.09 (0.09)	0.13 (0.11)
$\gamma_w$		0.05 (0.16)	0.12 (0.16)		0.11 (0.15)	0.15 (0.16)
$\gamma_m$		-0.20 (0.22)	-0.16 (0.25)		-0.49 (0.22)	-0.36 (0.26)
$R^2$	0.67	0.70	0.67	0.64	0.70	0.65
ABM $R^2$	0.69	0.69	0.66	0.68	0.72	0.67
HET	0.17	0.82	0.64	0.35	0.48	0.33

Estimation results for models 1, 2 and 3 detailed in the text. The specification for model 1 (M1) is  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \varepsilon_k$ , for model 2 (M2):  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \delta_d IRC_k^d + \delta_w IRC_k^w + \delta_m IRC_k^m + \gamma_d IRJ_k^d + \gamma_w IRJ_k^w + \gamma_m IRJ_k^m + \varepsilon_k$  and for model 3 (M3):  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \gamma_d IRJ_k^d + \gamma_w IRJ_k^w + \gamma_m IRJ_k^m + \varepsilon_k$ . The ABM  $R^2$  is the adjusted  $R^2$  measure as suggested by Andersen, Bollerslev, and Meddahi (2002). The HET line gives the P-values for the F-test of no heteroscedasticity. Newey-West standard errors are given in parenthesis. The time period is 1/2/1990 - 3/5/2003 for both indexes.

**Table 2**  
**Encompassing regressions: models 1, 2 and 3:  $h = 10$  days**

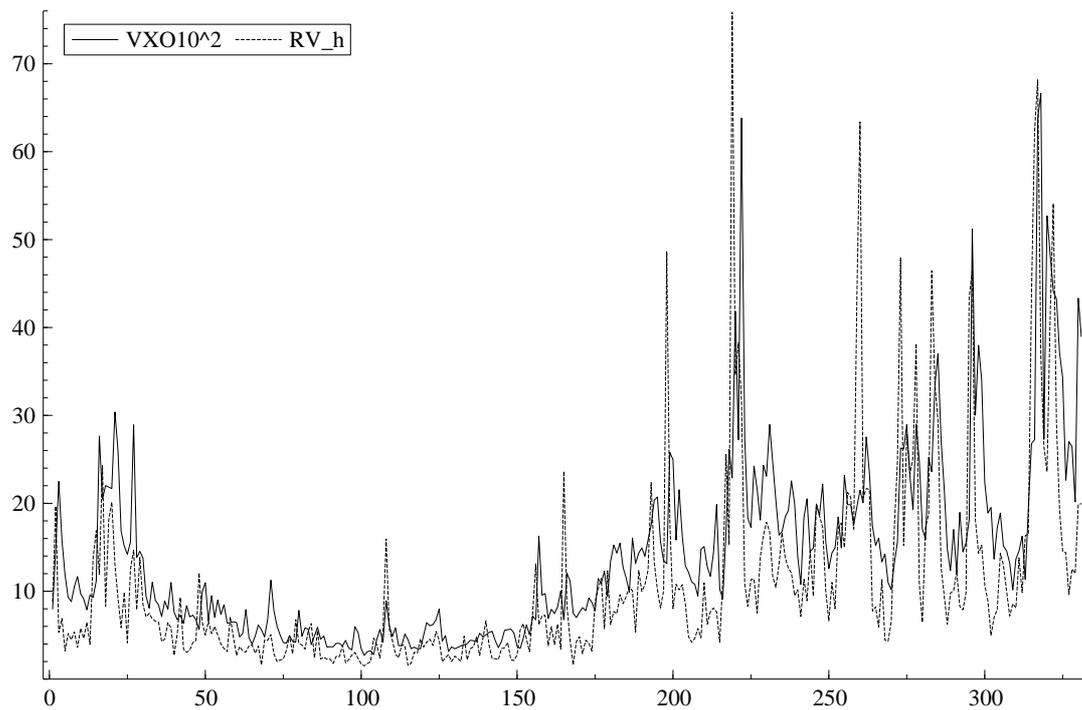
	S&P100 index			S&P500 index		
	M1	M2	M3	M1	M2	M3
$\beta_0$	-0.33 (0.08)	0.83 (0.20)	-0.32 (0.10)	-0.62 (0.12)	1.20 (0.18)	-0.63 (0.13)
$\beta_1$	0.98 (0.04)	0.56 (0.07)	0.97 (0.06)	1.06 (0.06)	0.43 (0.07)	1.07 (0.07)
$\delta_d$		0.06 (0.05)			0.05 (0.05)	
$\delta_w$		0.18 (0.08)			0.23 (0.09)	
$\delta_m$		0.17 (0.09)			0.29 (0.07)	
$\gamma_d$		0.10 (0.11)	0.07 (0.12)		0.15 (0.11)	0.15 (0.13)
$\gamma_w$		0.19 (0.22)	0.29 (0.21)		0.09 (0.24)	0.16 (0.23)
$\gamma_m$		-0.28 (0.24)	-0.21 (0.24)		-0.42 (0.21)	-0.27 (0.26)
$R^2$	0.69	0.72	0.69	0.65	0.71	0.66
ABM $R^2$	0.69	0.72	0.70	0.66	0.71	0.66
HET	0.16	0.98	0.73	0.49	0.95	0.81

Estimation results for models 1, 2 and 3 detailed in the text. The specification for model 1 (M1) is  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \varepsilon_k$ , for model 2 (M2):  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \delta_d lRC_k^d + \delta_w lRC_k^w + \delta_m lRC_k^m + \gamma_d lRJ_k^d + \gamma_w lRJ_k^w + \gamma_m lRJ_k^m + \varepsilon_k$  and for model 3 (M3):  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \gamma_d lRJ_k^d + \gamma_w lRJ_k^w + \gamma_m lRJ_k^m + \varepsilon_k$ . The ABM  $R^2$  is the adjusted  $R^2$  measure as suggested by Andersen, Bollerslev, and Meddahi (2002). The HET line gives the P-values for the F-test of no heteroscedasticity. Newey-West standard errors are given in parenthesis. The time period is 1/2/1990 - 3/5/2003 for both indexes.

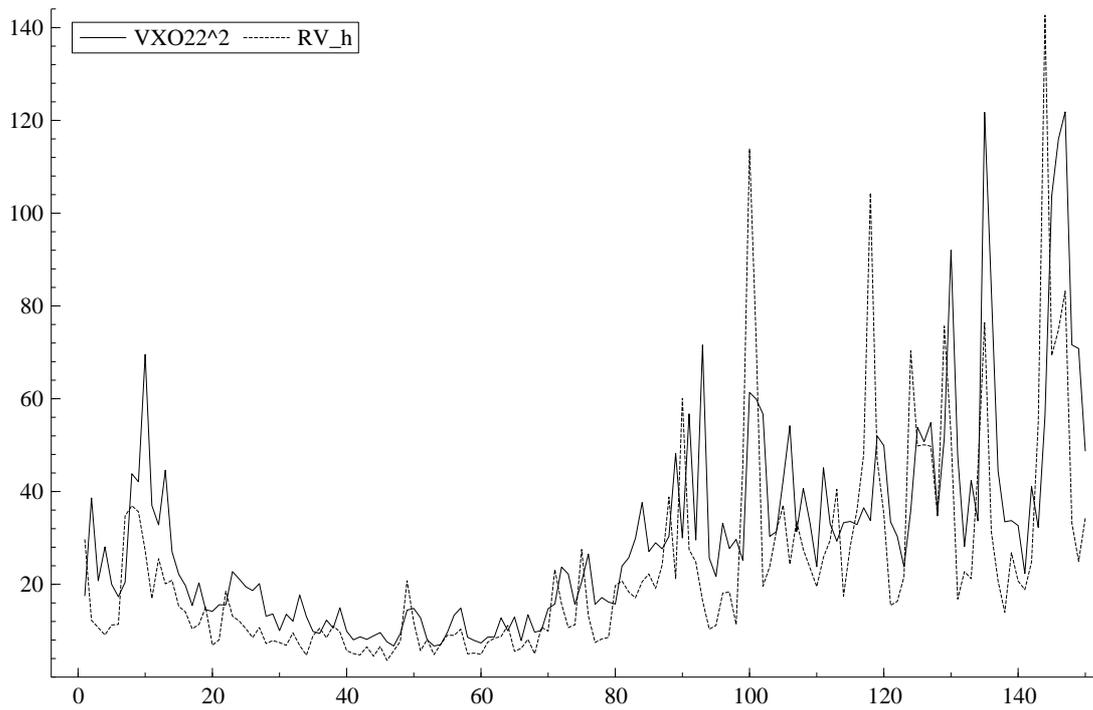
**Table 3**  
**Encompassing regressions: models 1, 2 and 3:  $h = 22$  days**

	S&P100 index			S&P500 index		
	M1	M2	M3	M1	M2	M3
$\beta_0$	-0.25 (0.12)	1.10 (0.61)	-0.23 (0.13)	-0.58 (0.18)	1.51 (0.48)	-0.66 (0.18)
$\beta_1$	0.96 (0.04)	0.60 (0.17)	0.97 (0.05)	1.05 (0.06)	0.49 (0.13)	1.09 (0.07)
$\delta_d$		0 (0.05)			0.10 (0.04)	
$\delta_w$		0.16 (0.10)			0.12 (0.10)	
$\delta_m$		0.19 (0.12)			0.29 (0.11)	
$\gamma_d$		0.03 (0.16)	0.02 (0.18)		0.11 (0.15)	0.01 (0.17)
$\gamma_w$		0.15 (0.37)	0.13 (0.38)		-0.24 (0.33)	-0.11 (0.34)
$\gamma_m$		-0.37 (0.25)	-0.22 (0.21)		-0.35 (0.21)	-0.31 (0.24)
$R^2$	0.69	0.72	0.69	0.67	0.72	0.67
ABM $R^2$	0.70	0.72	0.70	0.67	0.72	0.67
HET	0.01	0.49	0.07	0.10	0.86	0.46

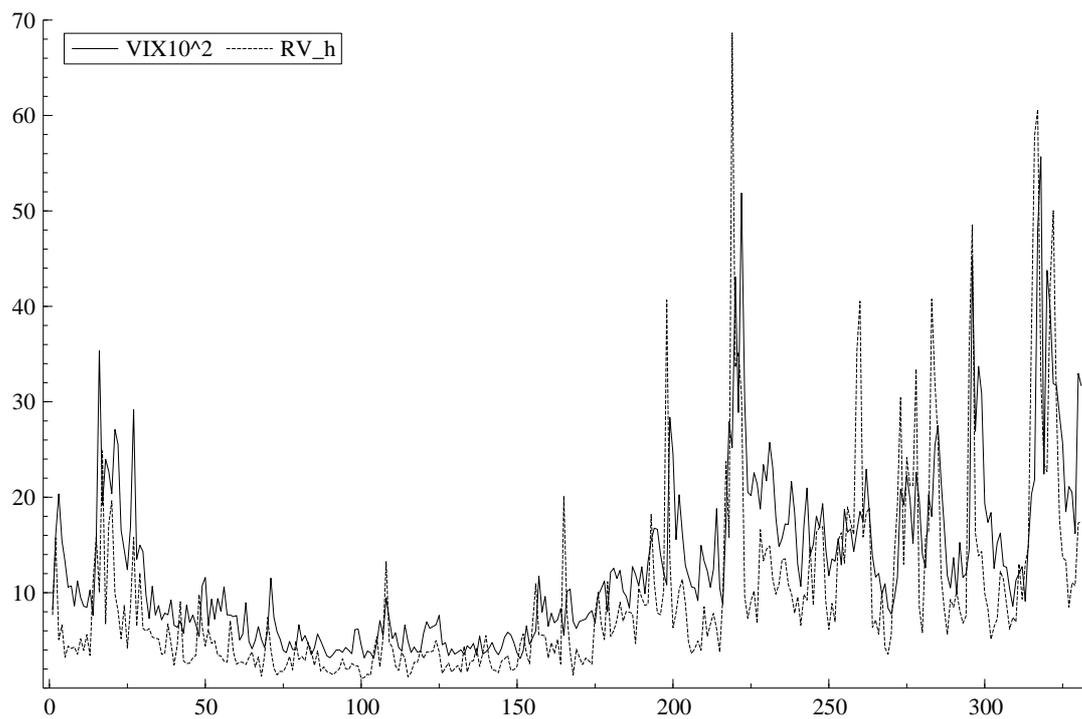
Estimation results for models 1, 2 and 3 detailed in the text. The specification for model 1 (M1) is  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \varepsilon_k$ , for model 2 (M2):  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \delta_d lRC_k^d + \delta_w lRC_k^w + \delta_m lRC_k^m + \gamma_d lRJ_k^d + \gamma_w lRJ_k^w + \gamma_m lRJ_k^m + \varepsilon_k$  and for model 3 (M3):  $IRV_k^h = \beta_0 + \beta_1 IIMP_k^h + \gamma_d lRJ_k^d + \gamma_w lRJ_k^w + \gamma_m lRJ_k^m + \varepsilon_k$ . The ABM  $R^2$  is the adjusted  $R^2$  measure as suggested by Andersen, Bollerslev, and Meddahi (2002). The HET line gives the P-values for the F-test of no heteroscedasticity. Newey-West standard errors are given in parenthesis. The time period is 1/2/1990 - 3/5/2003 for both indexes.



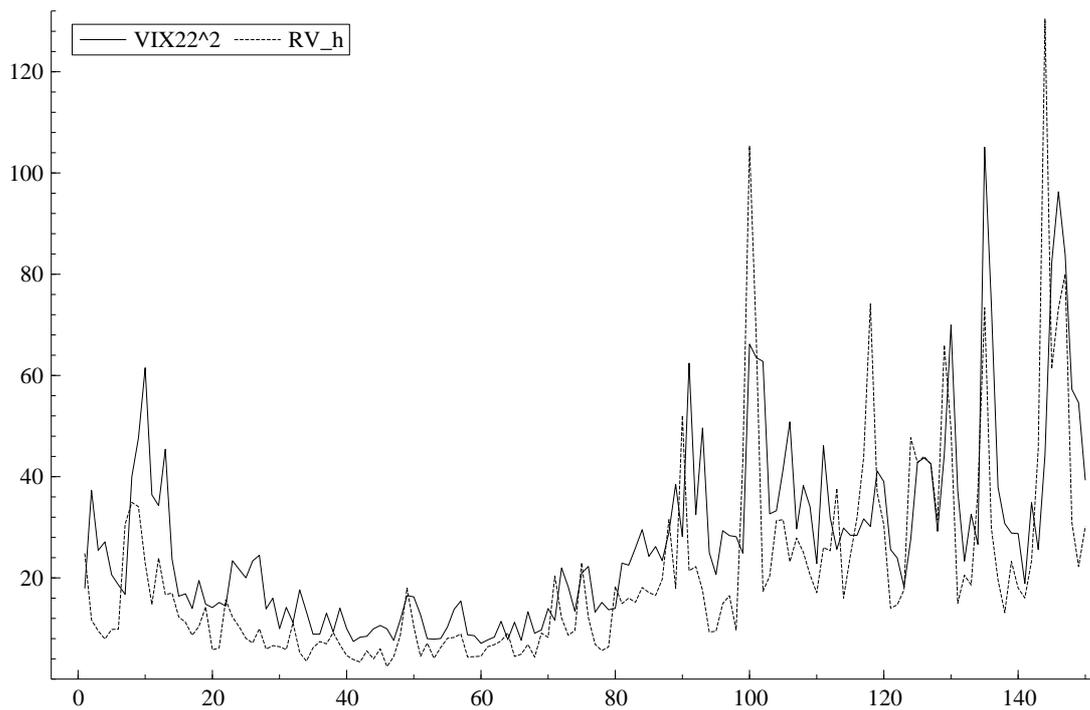
**Figure 1.** S&P100 index: implied volatility forecast at the 10-day horizon and subsequent 10-day realized volatility (RV\_h). The time period is 1/2/1990 - 3/5/2003 (non-overlapping data).



**Figure 2.** S&P100 index: implied volatility forecast at the 22-day horizon and subsequent 22-day realized volatility (RV\_h). The time period is 1/2/1990 - 3/5/2003 (non-overlapping data).



**Figure 3.** S&P500 index: implied volatility forecast at the 10-day horizon and subsequent 10-day realized volatility (RV\_h). The time period is 1/2/1990 - 3/5/2003 (non-overlapping data).



**Figure 4.** S&P500 index: implied volatility forecast at the 22-day horizon and subsequent 22-day realized volatility ( $RV_h$ ). The time period is 1/2/1990 - 3/5/2003 (non-overlapping data).