

Long-run volatility dependencies in intraday data and mixture of normal distributions*

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Abstract

In this paper, we study the behaviour of the long-memory in the return volatility using high-frequency data on the Deutschemark-US dollar. In particular, we provide evidence of the overestimation of the long-memory when we do not take into account the presence of jumps (outliers) in the series. After filtering the series from its seasonal pattern, and by using a mixture of normal distributions, the long-memory parameter is found to be constant across different sampling frequencies, highly reduced (compared to the normal distribution) but still significant.

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1 Introduction

The temporal dependence in the volatility is one of the most striking features of financial series recorded at various frequencies. Quite recently, a huge empirical econometric literature (see Granger and Hyung, 1999, and Mikosch and Stărică, 1999, among others), has been devoted to explain the long-memory behaviour of such a series as the result of neglecting structural change. On the contrary, according to Andersen and Bollerslev (1997a, 1998) and Andersen, Bollerslev and Cai (1999), the long-memory characteristic appears inherent to the intradaily return series because they manifest themselves rather than tied to infrequent structural shifts as suggested by Lamoureux and Lastrapes (1990). Diebold and Inoue (1999) provide, in the case of various simple models, an analytical proof that long-memory and structural change are easily confused and argue that “*even if the truth is structural change, long-memory may be a convenient shorthand description, which may remain useful for tasks such as prediction*”. In particular, they show that stochastic regime switching (for instance, mixture model, STOPBREAK, Engle and Smith, 1999 and Markov-Switching model, Hamilton, 1989) is observationally equivalent to long-memory, so long as only a small amount of regime switching occurs. The above mentioned stochastic regime switching models resemble a standard probability distribution that is called a *mixture of normal distributions* (see Jorion, 1988, Vlaar and Palm, 1993). In this respect, Beine and Laurent (1999) show that the long-memory parameter may be highly reduced (by one half) when modeling four daily exchange rates vis-a-vis the USD has being generated from a mixture of normal distributions (a bernoulli-normal distribution).

In this paper, we study the behaviour of the long-memory in the return volatility using high-frequency data on the Deutschemark-US dollar spot exchange rate (DM-USD).¹ The aim of the paper is both to provide evidence of the overestimation of the long-memory when we do not take into account the presence of jumps (outliers) in the series and to reinforce the argument that long-memory may be an intrinsic property of the exchange rate returns.

The intraday volatility process of the exchange rate is quite involved. New phenomena become visible as one proceeds from daily returns to intraday returns. Andersen and Bollerslev (1997a,b, 1998), Andersen, Bollerslev and Cai (1999) and Guillaume *et al.* (1995) interpret the overall volatility process as the simultaneous interaction of numerous independent volatility components: periodic volatility components (associated with calendar effects), short-run volatility components (associated with economic news) and longer-run volatility components (associated with persistent unobserved factors). As pointed out by Andersen and Bollerslev (1997b), it is necessary to pre-filter the data for its periodicity before estimating a (Fractionally Integrated) GARCH model.

In order to motivate the empirical relevance of these ideas, Figure 1 plots the lag 5 through 1440 sample autocorrelation for the five-minute absolute returns, $|R_{t,n}|$.²

¹A more detailed description and analysis of the data is contained in Appendix 1.

²Absolute returns are often used as a proxy of the volatility, see Ding *et al.* (1993).

INSERT FIGURE 1 ABOUT HERE

The daily periodicity phenomenon is apparent. Andersen and Bollerslev (1997b, 1998) propose an attractive methodology based on the Flexible Fourier Form (FFF) that allows a direct interaction between the level of the daily volatility and the shape of the intradaily pattern. Their model is a good starting point for high-frequency volatility modeling in a coherent framework. We apply their general framework with some differences: we take into account explicitly the Daylight Saving Time, only the US macroeconomic announcements are studied and the daily volatility component is calculated from a FIGARCH model. Given the estimates of the determinist periodicity effects, we filter the five-minute absolute returns to obtain an innovation process $|R_{t,n}|/\hat{s}_t$ that should be rid of periodicity effects. Details on this filtering procedure are proposed in the following sections. Figure 2 depicts the fifty days correlogram of filtered five-minute absolute returns, $|\tilde{R}_{t,n}|$.

INSERT FIGURE 2 ABOUT HERE

This figure shows a strictly positive and slowly declining correlogram. Spikes are apparent at the daily frequencies but they are minor and do not distort the overall pattern.³ The most striking feature is the initial rapid decay in the autocorrelations followed by an extremely slow rate of decay. This is confirmed by the high value of the Ljung-Box statistic (258450) at lag 1440 (five days). This correlation structure is not compatible with the standard GARCH process introduced by Bollerslev (1986). Instead, Figure 2 clearly suggests the presence of a long-memory process in the absolute returns which is consistent with the fractionally integrated long-memory volatility model proposed by Baillie, Bollerslev and Mikkelsen (1996). When applying the fractional differencing operator $(1 - L)^{0.4}$ to the filtered five-minute absolute returns, we observe that the autocorrelations display much less long term dependence (see Figure 3). The Ljung-Box statistic is reduced to 4307, which is lower than in the previous case and for first differenced data (16580).

INSERT FIGURE 3 ABOUT HERE

By applying semi-parametric tools on the same dataset, Andersen and Bollerslev (1998) find an evidence of long memory in the volatility and conclude that the long-memory characteristic appears *inherent* to the absolute return series. In this respect, we estimate the FIGARCH model for several observation frequencies (5-, 10-, 15-, 20-, and 30-minutes). Instead of first filtering the data and then changing the frequency, as proposed by Andersen and Bollerslev (1997b), we first change the frequency and then filter the series by using the corresponding filter. Our estimation results suggest that allowing for jumps in the series (especially in the variance), highly reduces the long-memory property of the series but reinforce the idea that the long-memory is an intrinsic property of the exchange rate returns. This is consistent with the empirical evidence on stock returns volatility provided by Granger and Hyung (1999) and with Diebold and Inoue (1999) warnings about *“the temptation to jump to conclusions of structural change producing spurious inferences on long memory”*.⁴

³The regularity of the correlogram in Figure 2 can be compared to those of similarly filtered absolute returns presented in Payne (1996) and Andersen and Bollerslev (1997b, 1998).

⁴See Diebold and Inoue (1999) p.25.

In others terms, both features are necessary to capture the short run dynamics of exchange rate volatility. Moreover, unlike the normal assumption, modelling the series as being generated from a mixture of normal distributions tends to stabilize the d parameter across different sampling frequencies.

The remainder of the paper is organized as follows. Section 2 and its subsections present Andersen and Bollerslev's method to filter the series from its intraday periodicity. Section 3 describes the FIGARCH model, the estimation methods and comments the results. Finally, section 4 concludes.

2 The intraday volatility

The volatility dynamics of intraday foreign exchange rate returns are involved. There are intraday volatility patterns, reflecting the daily activity cycle of the regional centers as well as weekend and Daylight Saving Time effects, the macroeconomic announcement effects (immediately following the release) and standard volatility clustering at the daily level. Thus, we assume that the volatility process is driven by the simultaneous interaction of numerous components which are described below.

2.1 Study of the different volatility components

2.1.1 Periodic volatility components

As Dacorogna *et al.* (1993) wrote: *"The behavior of a time series is called seasonal if it shows a periodic structure in addition to less regular movements"*. The foreign exchange (FX) market show strong seasonal effects caused by the presence of the traders in the three major markets according to the hour of the day, the day of the week and the Daylight Saving Times. The major movements of intradaily returns volatility can be attributed to the passage of market activity around the globe. The global FX market consists of three major markets: the Far East, Europe and North America. Figure 4 depicts the average absolute returns over the (288) five-minute intervals.

INSERT FIGURE 4 ABOUT HERE (see Appendix 2)

This intraday pattern is quite similar across all day of the week with discrete changes in quoting activity marking the opening and closing of business hours in the three major regional centers, all of which have their own activity pattern. The following hours can be used as indicative: the Far East is open from 21:00 GMT to 6:00 GMT, Europe trades between 7:00 GMT and 16:00 GMT and trading in North America occurs from 12:00 GMT to 21:00 GMT. Using the discussion of market opening and closures presented above, we explain the intraday seasonal volatility as follows. At 0:00 GMT, the Far Eastern market has already been trading for around three hours and market activity is high. From 0:00 GMT until about 3:00 GMT, activity levels and volatility remain high. The lunchtime in Tokyo (3:00 GMT- 4:45 GMT) is the point of the day corresponding to the most prominent feature of the series. Volatility drops sharply and regains its former value at about

5:00 GMT. Generally, Europe begins to contribute to activity at around 7:00 GMT as the Far Eastern market begins to wane: there is a small peak in volatility. During European lunch hours (11:30 GMT), both activity and volatility know a slight lull. The most active period of the day is clearly when both the European and North American markets are open (between 12:00 GMT and 16:00 GMT). Volatility starts to decline as first the European and then US markets wind down. At around 21:00 GMT, the Pacific market begins to trade again and the daily cycle is repeated after midnight. This intraday pattern is consistent with previous evidence reported in Müller *et al.* (1990), Dacorogna *et al.* (1993), Guillaume *et al.* (1994) and Andersen and Bollerslev (1997b, 1998).

An other intraday pattern often recognized in high frequency returns is day-of-the-week dependencies. Andersen and Bollerslev (1998), with the same data set, find that Monday appears the least volatile, while Thursdays and Fridays are the most volatile. Evidence has shown these effects to be the result of macroeconomic news announcements, which are released mainly on these two days (Harvey and Huang, 1991).

Daylight Saving Times (*DST*) has also an effect on the seasonal pattern. Indeed, *DST* changes will influence the local time relative to GMT and thus the intraday volatility pattern in reference to GMT. Both North America and Europe lose one hour relative to GMT in summer months. The Far Eastern local time remains unchanged. Andersen and Bollerslev (1998) and Payne (1997) studied the *DST* problem. Andersen and Bollerslev show that the volatility pattern appears translated leftward by exactly one hour between 6:00 GMT and 21:00 GMT during the US Summer Time regime.

The seasonal pattern, presented above, seems fully explainable. Failure to take account of those intradaily seasonals is likely to result in misleading statistical analyses. The first authors who have reported intraday analysis (Wasserfallen and Zimmermann, 1985, Feinstone, 1987, Ito and Roley, 1987, Wasserfallen, 1989 and Goodhart and Figliuoli, 1991) limited themselves to certain periods of the day, generally the most active ones for a particular market center, so the problem of daily and weekly seasonality was avoided.

The seasonal phenomena in the volatility of FX markets can be modeled in a variety of ways. Baillie and Bollerslev (1991) use a GARCH specification with seasonal dummy variables for modeling the conditional volatility on hourly forex returns on data from the first six months of 1986. For the current study, this would require estimating 288 time-of-day parameters, if one dummy variable were created for each five-minute interval. The number of variables required is very large and it is unlikely to be effective in capturing the complexity of the seasonal patterns. Another possibility to accommodate seasonality is to modify the traditional GARCH type models (Bollerslev and Ghysels, 1994). Alternatively, the market volatility can be tied to the intensity of trading via a subordinate stochastic process representation, as suggested by Clark (1973). This approach has been adopted in some recent works by researchers from Olsen & Associates (see for example, Dacorogna *et al.*, 1993, Müller *et al.*, 1992). Instead of modeling asset price behavior in calendar time, price movements can be represented as being driven by an information arrival process which itself evolves randomly with certain predictable patterns through time. In Dacorogna *et al.*, especially, the seasonal volatility patterns are modeled

by a new time scale, named v -time, under the assumption of three main geographical areas where most of the worldwide trading activity is centered: East Asia, Europe and America. Their time scale conversion expands periods with high average volatility and contracts those with low volatility. Their method smooths the seasonal pattern. Another strategy, the one used in this paper, is to seasonally adjust the data. We define the filtered return series $R_{t,n}/\widehat{s}_{t,n}$ where $\widehat{s}_{t,n}$ refers to the periodic intraday volatility component which may be modeled by different ways (see, for instance, Taylor and Xu, 1995, Chang and Taylor, 1996 and Andersen and Bollerslev, 1997b). The method, adopted in this paper, is the Flexible Fourier Form developed by Andersen and Bollerslev (1997b): intraday seasonality was modeled using several sinusoidal and quadratic parameters.⁵ The general formulation of the flexible Fourier form is the following:

$$f(t, n) = \mu_0 + \mu_1 \frac{n}{N_1} + \mu_2 \frac{n^2}{N_2} + \sum_{i=1}^P (\gamma_i \cos \alpha_i n + \delta_i \sin \alpha_i n) + \sum_{j=1}^2 \omega_j DST_j + \sum_{k=1}^D \lambda_k I_k(t, n) \quad (1)$$

where we consider the n -th interval⁶ in the t -th day, N is the number of intervals per day, $N_1 = (N + 1)/2$ and $N_2 = (N + 1)(N + 2)/6$ are normalizing constants and $\alpha_i = \frac{2\pi i}{N}$. As mentioned earlier, the DST alters the form of the seasonal. Therefore, we estimate two seasonal regimes: Summer Time⁷ and Transition period.⁸ Hence, there are two different dummy variables (DST_j) according the time of the year, $j = 1$ is the Transition Time period with $DST_1 = 1$ on this period and 0 otherwise ; $j = 2$ is the Summer Time period with $DST_2 = 1$ on this period and 0 otherwise.

The smooth seasonal generated from the Fourier terms is unlikely to cope well the sharp drop in volatility, for instance, around lunch in the Far East and the day of the week dependencies. To fill this gap, we add $\sum_{k=1}^D \lambda_k I_k(t, n)$ where $I_k(t, n)$ is an indicator variable for event k during interval n on day t . The events may be as well calendar effects as announcement effects (see next section). Following Andersen and Bollerslev (1998), we impose a reasonable declining weight structure on the volatility response pattern $\lambda(k, i) = \lambda_k \cdot \gamma(i)$, $i = 0, 1, 2, \dots, N_k$ where the pre-specified $\gamma(i)$ coefficients are determined by a specific polynomial and event k impacts volatility over N_k intervals. For the Tokyo open (0:00-0:35 GMT), we choose a linear volatility decay. The volatility decay pattern around the weekends (early monday morning (21:00-22:30 GMT), late friday (17:00-21:00 GMT, US Winter Time or 16:00-21:00 GMT, US Summer Time) is restricted to a second order polynomial.

⁵Payne (1997) uses a similar method in his stochastic variance model of the DM-USD exchange rate. Beattie and Fillion (1999) also use it to assess the effectiveness of Canada's official foreign exchange interventions on intraday volatility of the Can-USD exchange rate.

⁶For five-minute returns, n equals 144 at 12:00 GMT.

⁷ DST changes occurred in Germany and other European countries in the last weekend of March and September. In the US, changes occurred in the first weekend of April and last week of October. Japan did not have Daylight Saving Times changes.

⁸Between the last weekends of September and October, the USA is still in Summer Time, but Europe is already in Winter Time. This period lasts 4 weeks. In the week before the first weekend in April, the USA is still in Winter Time but Europe is already in Summer Time.

2.1.2 Short-run volatility components: macroeconomic announcements effects

Macroeconomic announcements are relevant for proper modeling of the volatility process. Indeed, Ederington and Lee (1993) showed that the largest returns appear linked to the release of public information (in particular, certain macroeconomic announcements). Studies that examine the impact of scheduled news announcements on high frequency volatility are various (for instance, Andersen and Bollerslev, 1998, Ederington and Lee, 1993, 1995, Goodhart *et al.* 1993, Harvey and Huang 1991, Ito and Roley, 1987, DeGennaro and Shrieves, 1997 and Payne, 1997). The findings of these studies are consistent, indicating that the releases induce quite dramatic price adjustments but the associated volatility shocks appear short-lived.⁹ These studies are also interesting to measure of the significance that the market attributes to each type of announcement.

We can get a precise economic impact by using the forecast errors associated with announcements (Almeida *et al.*, 1996 and Payne, 1997). The forecast errors are created as the difference between the actual announced figure and a median survey expectation. We can also get the general impact of announcements by using a simple dummy specification for announcements. Our analysis focuses on a set of monthly, US, macroeconomic announcements. These announcements are all related to the real economy. It consists of the Employment Report, the Merchandise Trade Deficit, the Producer Price Index, Durable Goods Orders, Retail Sales, Housing Starts, Leading Indicators, Industrial Production and Capacity Utilization¹⁰, Consumer Price Index, Consumer Confidence Index, NAPM survey and Gross Domestic Product (GDP). The category of news is extracted from the Reuters news items¹¹ using various keyword combinaisons.

In equation (1), the $I_k(t, n)$ indicators allows for the inclusion of either regular dummy variables or a pre-specified volatility response pattern associated with a calendar related characteristic or news macroeconomic announcements effects. The effect of news on volatility before announcement is not studied here. However, if announcements affects volatility for a hour, there are 13 separate event-specific coefficients to estimate. Given the limited number of occurrences of each type of news announcement, it is not possible to simultaneously estimate separate coefficients for each event and time interval following the news releases. Instead, following Andersen and Bollerslev (1998), we impose a reasonable declining weight structure on the volatility response pattern. The response pattern following each of the announcements is approximated by a third-order polynomial restricted to reach zero at the end of the one hour response horizon. The dynamic response pattern is $\lambda(k, i) = \lambda_k \cdot \gamma(i)$, $i = 0, 1, 2, \dots, 12$, where the pre-specified $\gamma(i)$ coefficients are determined by a third-order polynomial and λ_k is the announcement specific loading coefficient.

⁹There are signs of higher volatility for several hours following the announcement.

¹⁰The Industrial Production and the Capacity Utilization are announced together.

¹¹The O&A data also include all of the news headlines that appeared on the Reuters money news-alerts screens. As with the quotations, these are time stamped to the second in GMT and constitute the basis for our analysis of announcement effects. Comparison of the time stamps for scheduled news releases with the known release schedules indicates that Reuters is timely with respect to scheduled news. During the sample period, a total of 105065 such headlines appeared.

2.1.3 Daily volatility components

Numerous studies suggest that daily and monthly foreign exchange returns exhibit significant volatility clustering. Thus, these ARCH effects at lower frequencies cannot exist exclusively at these frequencies as the aggregation of intraday returns would not be able to accommodate the persistent volatility processes present at the daily level. It is necessary that the low-frequency volatility embodied in high frequency data has to be modeled. Moreover, Andersen and Bollerslev (1998) demonstrated that daily GARCH volatility predictions are strongly related to the sum of the absolute intraday changes in the foreign exchange for the following day. Indeed, they noted that the correlation between the two series is 0.672, or an R -squared of $(0.672)^2=45.2\%$. So, to take into account the daily component of foreign exchange volatility, we used a daily volatility forecast $(\hat{\sigma}_t)$.¹²

As Andersen and Bollerslev (1998) write: *“Unfortunately, most empirical work has studied each of the above phenomena - the intraday and intraweekly patterns (calendar effects), the announcements (public information effect), and the interday volatility persistence (ARCH effects) - in isolation. This is ultimately not satisfactory”*. Indeed, earlier studies tend to emphasize one of the following three components. Recent findings suggest that the three factors should be accounted for simultaneously to capture the overall intraday pattern.

2.2 Modeling simultaneously the systematic components of volatility

Andersen and Bollerslev (1997b) proposed a method based on the Flexible Fourier Form (FFF) to model the intraday volatility periodicity, the effects of macroeconomic news announcements and the persistent daily volatility dependencies found in foreign exchange data. We apply their framework which consists in decomposing the five-minute returns $(R_{t,n})$ as:

$$R_{t,n} - \bar{R}_{t,n} = \sigma_{t,n} \cdot s_{t,n} \cdot Z_{t,n} \quad (2)$$

where $\bar{R}_{t,n}$ is the expected five-minute return, $\sigma_{t,n}$ is a daily volatility factor, $s_{t,n}$ represents both the calendar features and the macroeconomic announcement effects and $Z_{t,n}$ is an i.i.d. mean zero and unit variance innovation term. In order to obtain an operational regression equation, Andersen and Bollerslev propose to impose some restrictions and some additional structure (see Andersen and Bollerslev, 1997b for more details). We estimate the following operational regression:¹³

¹²This daily volatility is obtained by estimating an AR(1)-FIGARCH(1, d ,1) over the period January 1980 to September 1993.

¹³The FFF estimation involves a two-step procedure (Andersen and Bollerslev, 1997).

$$\begin{aligned}
2 \ln \frac{|R_{t,n} - \bar{R}|}{\hat{\sigma}_t / N^{1/2}} &= c + \mu_0 + \mu_1 \frac{n}{N_1} + \mu_2 \frac{n^2}{N_2} + \sum_{i=1}^P (\gamma_i \cos \alpha_i n + \delta_i \sin \alpha_i n) \\
&\quad + \sum_{j=1}^2 \omega_j DST_j + \sum_{k=1}^D \lambda_k I_k(t, n) + \hat{u}_{t,n}
\end{aligned} \tag{3}$$

where \bar{R} denotes the sample mean of the five-minute returns, $c = E(\log Z_{t,n}^2) + E(\log \sigma_{t,n}^2 - \log \hat{\sigma}_{t,n}^2)$ and $\hat{u}_{t,n}$ is the error term which is stationary. $\hat{\sigma}_t$ is an estimate of the daily volatility component. The daily volatility component is $\hat{\sigma}_{t,n} = \hat{\sigma}_t / N^{1/2}$ where $\hat{\sigma}_t$ is derived from a daily AR(1)-FIGARCH(1, d , 1) model. All coefficients are estimated simultaneously (absolute t-statistics are robust for heteroskedasticity). The estimation results relative to the five-minute returns are reported in Table 1.

TABLE 1: Results of the FFFestimation on five-minute returns

μ_0+c	-10.0708	6.151
μ_1	13.6421	2.890
μ_2	-4.3987	2.864
γ_1	1.9814	2.131
γ_2	0.2966	1.556
γ_3	0.4883	6.601
γ_4	0.2621	3.737
γ_5	0.2407	3.799
γ_6	-0.0304	1.229
δ_1	0.4203	2.630
δ_2	0.5181	5.801
δ_3	-0.1062	2.402
δ_4	0.1032	3.255
δ_5	0.2156	5.897
δ_6	0.1607	3.426
ω_1	-0.1961	0.810
ω_2	0.0243	0.192
Tokyo opening	1.4236	4.303
Tokyo lunch 1	-1.3415	9.551
Tokyo lunch 2	-0.1036	3.127
Monday 1	0.2985	1.185
Monday 2	0.0471	0.776
Friday late 1	-0.0009	0.004
Friday late 2	-0.0355	1.804
Tuesday	0.2471	2.697
Wednesday	0.2606	2.505
Thursday	0.2681	2.378
Friday	0.0994	0.549
Consumer Confidence	0.4722	1.374
Consumer Price Index	1.1652	4.356
Capacity Utilization Industrial production	0.0391	0.167
Durable Goods Orders	1.5073	4.788
Index of Leading Indicators	0.2691	1.524
US NAPM survey	-0.1448	0.337
Housing starts	0.6387	1.972
Producer Price Index	0.4658	1.099
Advance Retail Sales	0.5657	1.337
Merchandise Trade Balance	1.2874	3.232
GDP	1.3731	5.501
Jobless Rate	2.7441	8.218

Robust absolute t-statistics are reported in the third column.

After some experimentation, we found that $P = 6$ is sufficient to capture the basic shape of the series. This FFF provides an estimated seasonal pattern that fit reasonably well the intraday periodicity. All coefficients associated with the simple Fourier form are significant, except for the second and the last cosine terms. The Tokyo market opening effect is captured by a single coefficient (it allows for a linear decay in the associated volatility burst). We note a strong market opening effect. Indeed, it has an immediate response coefficient of 1.42 implying that volatility jumps by 142 percent at 9 a.m. Tokyo time. The assessment of the remaining calendar and announcement effects is more complicated because the regressors are not simple

indicators, but imply pre-specified dynamic response patterns. For instance, the Tokyo lunch and the Friday late effects are accommodated by a second-order polynomial over the corresponding intervals, resulting in two regression coefficients (Tokyo lunch 1, Tokyo lunch 2, Friday late 1..., see Table 1) for each period. Besides, we note that the Tokyo lunch exerts a considerable effect. For the announcements, we use a third-order polynomial to capture their impact on the volatility. The actual estimates for this polynomial is given by $\gamma(i) = 1.9228 [1 - (i/13)^3] - 0.7205 [1 - (i/13)^2] i + 0.0988 [1 - (i/13)^3] i^2$. Hence, the instantaneous jump in the volatility equals $\exp(\lambda_k \cdot \gamma(0)/2) - 1$. In particular, the instantaneous jump for the Jobless Rate equals $\exp(2.7441 \cdot (1.9238/2)) - 1 = 1.63$ or 163%. By the way, the response at the i^{th} lag equals $\exp(\lambda_k \cdot \gamma(i)/2)$. Table 1 reports estimates of separate λ_k coefficients for each type of announcement. The Jobless Rate clearly has the greatest effect on volatility, the coefficient $\lambda_{jobless}$ being the highest. The next most important announcements are the GDP, the Merchandise Trade Balance, the the Durable Goods Orders and the Consumer Price Index. The Consumer Confidence, the Housings starts, the Producer Price Index and the Advance Retail Sales figures form a medium impact sub-group. Finally, there is a group of low impact announcements which comprises the Capacity utilization/Industrial Production, the Index of Leading Indicators and the US NAPM survey. In the regression, we incorporated day-of-the week dummies for all weekdays except Monday. There is a clear distinction between midweek days and Mondays and Fridays. However, both the Monday morning and the Friday afternoon effects are insignificant.

Following Andersen and Bollerslev (1997b), the link between $\hat{s}_{t,n}$ and $\hat{f}(t,n)$ is as follows:

$$\hat{s}_{t,n} = \frac{T \exp(\hat{f}(t,n)/2)}{\sum_{t=1}^T \sum_{n=1}^N \exp(\hat{f}(t,n)/2)}$$

where $\hat{s}_{t,n}$ is the estimator of the intraday periodic component for interval n on day t . Figure 5 shows the average one-day estimated seasonality ($\hat{s}_{t,n}$) of the five-minute returns.

INSERT FIGURE 5 ABOUT HERE

3 Long-memory from intraday returns

Quite recently, Andersen and Bollerslev (1997b) stressed the danger of estimating GARCH models on high-frequency data without removing its intraday pattern. After applying the FFF on the raw data, Figures 2 and 3 clearly suggest the presence of long-memory in the volatility of the filtered DM-USD, which is became a stylised fact in the empirical literature. It is well known that the degree of fractional integration should be identical across different sampling frequencies under quite general distribution assumption (see Andersen and Bollerslev, 1997a and Bollerslev and Wright, 1998). From frequency-domain methods, Andersen, Bollerslev and Cai (1999) estimate the degree of fractional integration for the 5-, 10-, 15- and 30-minute absolute Nikkei 225

returns (from January 2, 1994 to December 31, 1997). The d parameters are respectively 0.429, 0.404, 0.482 and 0.485 which are indistinguishable and they conclude that *the long-memory feature is an inherent property of the Nikkei 225 volatility*.¹⁴

The aim of this section is to provide a parametric estimation of the long-memory property. The FIGARCH $(1, d, 1)$ model proposed by Baillie *et al.* (1996) is given by the two following equations:

$$\tilde{R}_{t,n} = \mu + \rho \tilde{R}_{t,n-1} + \epsilon_{t,n}, \quad \epsilon_{t,n} | \Omega_{t,n} \sim D(0, \sigma_{t,n}^2) \quad (4)$$

$$\sigma_{t,n}^2 = \omega + \left[1 - (1 - \beta_1 L)^{-1} (1 - \phi_1 L) (1 - L)^d \right] \epsilon_{t,n}^2 \quad (5)$$

where an AR(1) process is allowed for $\tilde{R}_{t,n}$, μ is the mean of the process and $\Omega_{t,n}$ is the information set at time t and interval n . $\rho, \mu, \omega, \beta_1, \phi_1$ and d are parameters to be estimated with d being the fractional integration parameter and finally, L is the lag operator.¹⁵ For a FIGARCH(1, d , 1), sufficient conditions for the conditional variance to be strictly positive are given in Baillie *et al.* (1996).¹⁶ For higher orders, these conditions are cumbersome to derive, which obviously hampers the generalisation of the FIGARCH specification to higher orders. Interestingly, the FIGARCH(1, d , 1) model nests the GARCH(1,1) model (Bollerslev, 1986) for $d = 0$ and the IGARCH model (Engle and Bollerslev, 1986) for $d = 1$. As advocated by Baillie *et al.* (1996), the IGARCH process may be seen as too restrictive as it implies infinite persistence of a volatility shock. Such a dynamics is contrary to the observed behavior of agents and does not match the persistence observed after important events (see Baillie *et al.*, 1996, Bollerslev and Engle, 1993). By contrast, for $0 < d < 1$, the FIGARCH model implies a long-memory behavior, i.e. a slow decay of the impact of a volatility shock.

The first candidate distribution (D) for the estimation of this model is the normal one. In the Gaussian case, the log-likelihood of the model takes the following form:

$$Ln(L_{Norm}) = \sum_{t=1}^T \sum_{n=1}^N \left[-0.5 \ln (2\pi\sigma_{t,n}^2) + (\epsilon_{t,n}^2/\sigma_{t,n}^2) \right] \quad (6)$$

where T and N are respectively the number of days and the number of intervals per day.

Recent developments in time series econometrics have been concerned with the interaction between structural change and long-memory. Diebold and Inoue (1999) show that stochastic regime switching may be observationally equivalent to long-range dependence. The key idea developed by these authors is that regardless of the sample size, long-memory can be detected if realizations tend to have just a few breaks. Granger and Hyung (1999)

¹⁴Notice that they also find $d = 0.476$ on a longer time series of daily Nikkei 225 absolute returns.

¹⁵We follow Baillie *et al.* (1996) and truncate the infinite Taylor approximation of $(1 - L)^d$ at a number of lags equal to 1000. Chung (1999) proposes an alternative specification of the FIGARCH due to the strong relationship between ω and the truncation order. We do not tackle this issue in this paper because the parameter of interest is d , which is not affected by this choice, as shown by Chung (1999).

¹⁶Some of these sufficient conditions are nevertheless not necessary. For instance, they specify $\omega > 0$. By contrast, our estimation procedure allows ω to be negative but checks the positiveness of the conditional variance on a case-by-case basis (see Nelson and Cao, 1992).

underline the fact that correcting for outliers may significantly decrease the long-memory parameter. To cope with this issue, we use a mixture of normal distributions (bernoulli-normal) that allows for the possibility of endogeneously determined jumps. Mixtures have been introduced in econometrics by Jorion (1988) and more recently adapted to the GARCH framework (with weekly data) by Vlaar and Palm (1994), Neely (1999) and to the FIGARCH (with daily data) by Beine and Laurent (1999). The major findings of these papers is that the volatility persistence significantly decreases when accounting for jumps in the series. This invalidates the IGARCH model, which is a common result in the empirical literature.

Considering this distribution, equation (4) can be rewritten as follows:

$$\tilde{R}_{t,n} = \mu + \lambda\tau + \rho\tilde{R}_{t,n} + \epsilon_t \quad (7)$$

where λ is the probability of a jump and τ is the size of the jump, while equation (5) remains unchanged. The log-likelihood then takes the following form:

$$\begin{aligned} Ln(L_{Bern-Norm}) = & -\frac{T}{2} \ln(2\pi) + \sum_{t=1}^T \ln \left\{ \frac{(1-\lambda)}{\sigma_{t,n}^2} \exp \left[\frac{-(\epsilon_{t,n}^2 + \lambda\tau)^2}{2\sigma_{t,n}^2} \right] \right. \\ & \left. + \frac{\lambda}{\sqrt{\sigma_{t,n}^2 + \delta^2}} \exp \left[\frac{-(\epsilon_{t,n}^2 - (1-\lambda)\tau)^2}{2(\sigma_{t,n}^2 + \delta^2)} \right] \right\} \end{aligned} \quad (8)$$

where δ^2 is the variance of the jump size.

Maximum likelihood estimations have been conducted for five frequencies, 5-, 10-, 15-, 20- and 30-minute returns for the normal and the bernoulli-normal distributions.¹⁷ Andersen and Bollerslev (1997b) first filter the five-minute returns ($R_{t,n}$) and then estimate the GARCH models on this filtered series ($\tilde{R}_{t,n}$). The method used to change the frequency from 5 to $5 * k$ minutes ($k = 1, 2, \dots$) is straightforward: $R_{t,n}^{(k)} = \sum_{i=(n-1)k+1, nk} R_{t,i}$, where $t = 1, 2, \dots, T$, $n = 1, 2, \dots, N$ and $N = 288/k$. By the same way, they calculate the filtered $5 * k$ minute returns as $\tilde{R}_{t,n}^{(k)} = \sum_{i=(n-1)k+1, nk} \tilde{R}_{t,i}$. In order to avoid an aggregation problem, we propose to change the frequency and after filter the series (finding the FFF relative to the frequency of interest). Details concerning this choice are reported in Appendix 2.

Results of the estimations are given in Tables 2 and 3:

¹⁷Intraday returns are very small values. For instance, the mean of the filtered five-minute returns equals 1.6×10^3 . To avoid convergence problems, we multiplied the returns by 10^5 . All the computations have been done in GAUSS 3.2 and Maxlik 4.0. A Gauss procedure to compute equation (5) is available at the following url: <http://www.egss.ulg.ac.be/econometrie/FIGARCH.SRC>

Table 2: AR(1) - FIGARCH(1,d,1) with the normal distribution

	5-min	10-min	15-min	20-min	30-min
μ	-0.6231 [0.532]	-1.5974 [0.678]	-0.1238 [0.035]	-1.7981 [0.372]	1.1771 [0.161]
ρ	-0.0981 [26.084]	-0.0983 [17.675]	-0.1035 [15.423]	-0.0886 [11.155]	-0.0729 [7.218]
$\omega \times 10^{-4}$	1.9519 [46.928]	4.0851 [33.743]	7.4404 [33.016]	10.5540 [27.074]	18.2861 [23.662]
d	0.4252 [60.604]	0.4009 [43.411]	0.3332 [37.232]	0.2879 [29.970]	0.2607 [26.090]
β_1	0.7109 [157.351]	0.6634 [98.022]	0.5811 [46.916]	0.3582 [8.156]	0.2265 [5.044]
ϕ_1	0.5401 [78.490]	0.4653 [43.719]	0.4227 [29.995]	0.2237 [5.516]	0.1236 [2.833]
b_3	0.3470 ***	0.0988 ***	0.0634 ***	-0.0322 **	0.1767 ***
b_4	22.1597 ***	9.8128 ***	8.2316 ***	4.0885 ***	6.0243 ***
AIC	14.9607	15.5806	15.9550	16.1788	16.5811
SBIC	14.9615	15.5819	15.9570	16.1813	16.5846
$S(1)$	0.0056	0.1152	4.2131 **	1.6411	0.3304
$\text{Log Lik} \times 10^{-4}$	-56.2272	-29.2777	-19.9871	-15.2002	-10.3849

Absolute t-statistics of maximum likelihood estimates are in brackets.

Statistics are computed on normalized residuals.

b_3 and b_4 are excess skewness and kurtosis.

AIC and SBIC are Akaike and Schwarz Bayesian information criteria.

** and *** indicate that the statistic is significant at 5 and 1%, respectively.

The data have been multiplied by 10^5 .

Table 3: AR(1) - FIGARCH(1,d,1) with the bernoulli-normal distribution

	5-min	10-min	15-min	20-min	30-min
μ	3.1289 [2.531]	3.1848 [1.257]	3.9089 [1.062]	7.8521 [1.559]	1.1855 [0.161]
ρ	-0.1078 [27.351]	-0.1018 [18.335]	-0.1081 [15.965]	-0.0984 [12.521]	-0.0846 [8.743]
ω	-3.2143 [0.006]	-4.4563 [0.002]	-1.7655 [0.000]	0.4062 [0.000]	4.5819 [0.000]
d	0.1159 [27.371]	0.1272 [19.940]	0.1204 [18.250]	0.1270 [16.214]	0.1460 [12.002]
β_1	0.8560 [164.243]	0.8884 [66.507]	0.9702 [166.127]	0.9748 [115.803]	0.8319 [11.226]
ϕ_1	0.9131 [232.678]	0.9099 [80.593]	0.9770 [216.450]	0.9796 [141.426]	0.8176 [10.463]
λ	0.1061 [37.840]	0.1231 [27.068]	0.1061 [20.4697]	0.1235 [16.102]	0.1261 [15.265]
τ	-32.4123 [2.641]	-23.9715 [1.130]	-29.8268 [0.854]	-68.9173 [1.710]	13.1748 [0.220]
$\delta^2 \times 10^{-5}$	5.8670 [130.685]	10.0062 [63.925]	16.6840 [41.3787]	17.1796 [25.878]	29.1330 [25.781]
b_3	-0.0338 ***	-0.0200	-0.0107	-0.0132	0.0276
b_4	0.4658 ***	0.5226 ***	0.5878 ***	0.3588	0.3895 ***
AIC	14.8607	15.4903	15.8560	16.0977	16.4781
SBIC	14.8618	15.4923	15.8589	16.1015	16.4834
$S(1)$	2.1986	0.8726	1.4177	0.2187	0.0001
$\text{Log Lik} \times 10^{-4}$	-55.8509	-29.1076	-19.8627	-15.1237	-10.3201

Note: See Table 2.

Several comments are in order.

First, the bernoulli-normal distribution seems appropriate to describe the series. Likelihood ratio tests (LRT), not reported here, and the information criteria clearly favour the bernoulli-normal distribution for each sampling frequencies. Looking at the bernoulli parameters reveals that the probability of a jump is about 10%. Interestingly, as in Beine and Laurent (1999, 2000), we can distinguish two *regimes* (distributions): a low volatility regime and a high volatility regime (τ and δ^2 being respectively highly non significant and significant).

Second, relying on the normalized residuals (see Vlaar and Palm, 1994), all excess kurtosis (b_4) are found to be significant at the 5% level while there is no excess skewness (b_3). Nevertheless, b_4 turns out to be much lower than those obtained for the normal distribution, which confirms that accounting for a non uniform flow of information reduces excess kurtosis. To test for possible remaining ARCH effects, we use the rank test proposed by Wright (1998) that it is more powerful than alternative tests when the residuals are highly non normal (something we suspect here).¹⁸ According to this statistic, the three models correctly account for the cluster of volatility phenomena for both distributions.

Third, concerning the constance of the d parameter, the normal and the bernoulli-normal distributions lead to different results. While d highly decreases when the sampling frequency decreases in the normal case (from 0.42 to 0.26), d turns around 0.12 in the later case.¹⁹ Quite interestingly, with the normal distribution, ω evolves in the opposite way of the d parameter (and is always significant). Similarly, δ^2 increases when the sampling frequency decreases in the bernoulli-normal case (and is always significant) but ω is not significant for the five frequencies.²⁰ So, there is a strong relation between the long-memory parameter and the variance of the jump size (δ^2) and neglecting the presence of outliers may lead to an overestimation of the long-memory behaviour.

4 Conclusion

As pointed out by Andersen, Bollerslev and Cai (1999), *“it remains an open issue to identify the specific economic forces that may generate the long-run persistence patterns. At an abstract level, one possibility is that it may arise from the interaction of a large number of heterogenous information arrival processes”*.

According to Andersen and Bollerslev²¹ (1997a, 1998) and Andersen, Bollerslev and Cai (1999), the long-

¹⁸The nonparametric rank test introduced by Wright (1998) can be used as a misspecification test suitable for GARCH and FIGARCH models. For fixed l , the test statistics $S(l)$ is given by $S(l) = T \sum_{i=1}^l \rho(s_{1t}, s_{1t-i})^2$ where $\rho(.,.)$ denotes the sample autocorrelation function and s_{1t} is given by $s_{1t} = (r(e_t^2) - \frac{T+1}{2}) / \sqrt{\frac{(T-1)(T+1)}{12}}$ where e_t are the standardised (here, normalized) residuals and $r(e_t)$ is the rank of e_t among e_1, e_2, \dots, e_T . Under the null of a correct specification in the conditional variance, Wright (1998) proposes to use a $\chi^2(l)$ distribution (the test is not perfectly exact). Results are only reported for $l = 1$ but are consistent with other values of l (5, 10 and 20 for instance).

¹⁹By estimating a Markov-Switching FIGARCH(1, d , 0), Beine and Laurent (2000) find $d = 0.09$ on a longer sample of daily DM-USD exchange rate returns (while the standard FIGARCH(1, d , 0) lead to a $d = 0.27$).

²⁰Chung (1999) proposes a different specification of the FIGARCH model, more in line with the ARFIMA model. Equivalence between the two specifications requires that $\omega = 0$. Our results suggest that the normal distribution fails to accept this restriction. Chung (1999) interprets this positiveness of ω (whose theoretical value is zero) as an artefact of the subjective choices of the truncation order of $(1 - L)^d$. However, the bernoulli-normal distribution leads to a different conclusion, ω being always non significant. By the way, we argue that finding $\omega > 0$ may also be due to the choice of the distribution and using a more appropriate distribution (that takes into account the presence of outliers) may overcome this problem.

²¹Andersen and Bollerslev (1997a) developed a theoretical framework which is built on the idea that the aggregate market

memory characteristic appears inherent to the return series because they manifest themselves, even over shorter time spans. They concluded that the source of fractional integration in the volatility is related to the data generating process itself, rather than tied to infrequent structural shifts as suggested by Lamoureux and Lastrapes (1990). By using parametric estimations, we also find evidence of long-memory in the DM-USD. However, after allowing for jumps in the series (especially in the variance), we conclude in favour of less long-memory than Andersen, Bollerslev (1997a) even if we reinforce their argument that long-memory is an intrinsic property of the exchange rate returns.

>From our results, we can argue that the volatility of the DM-USD describes the same long-memory behaviour across different sampling frequencies. However, accounting for jumps in the series (or for the presence of outliers) highly reduces this long-memory, which remains relevant at any significance levels. While d ranges from 0.26 to 0.42 in the normal case, d ranges from 0.11 to 0.14 in the bernoulli-normal one. This result is in line with the work of Beine and Laurent (2000) who find a large decrease of the d parameter (but still significant) when modeling daily DM-USD by a Markov-Switching FIGARCH to account for the possible structural change.²²

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volatility represents the manifestation of numerous heterogenous information arrival processes (some with short-run volatility dependencies and others possessing more highly persistent patterns). When time passes, the short-run processes decay significantly while the more persistent processes remain influential.

²²Unlike the Bernoulli-Normal distribution, Markov-Switching models can be viewed as time-varying mixtures. They are more flexible but also more computationally demanding, which make their use unattractive with large database (remember that we have more than 75000 observations for five-minute returns).

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Appendix 1: Data construction

The exchange rate data are tick-by-tick observations on the German mark price of the US dollar (DM-USD) as displayed on the Reuters FAFX screen from October 1, 1992, through September 30, 1993. There were 1472241 quotations in this year. Each quote contains a bid and an ask price along with the time to the nearest even second. Moreover, we utilize a daily time series of 3586 spot DM-USD exchange rates from January 3, 1980 through September 30, 1993. In this paper, only the bid prices are used because bid price is quoted in its entirety by Reuters. As noted by Dacorogna *et al.* (1993) and Zhou (1996), only the last two or three digits of the ask price are quoted. Note that the recording events j (for which the times are marked by t_j) are unequally spaced. As we are investigating the time series using equally spaced time intervals, we have to find a mapping procedure to fixed time steps, which are denoted by t_i . Our time steps are defined by using time intervals of $\Delta t = 5$ minutes length.²³ We applied a linear interpolation²⁴ as an appropriate method for interpolating the prices between the previous t_{j-1} and the next t_j data record surrounding the time step t_i with $t_{j-1} < t_i < t_j$. In general, a trader is not interested in the price, rather they are interested in the return that they will gain from that investment. Statistically speaking looking at the raw prices is not very constructive, as the prices can be highly correlated and in general are not stationary. The n th return within day t , $R_{t,n}$, can be defined as the change in the logarithms of prices: $R_{t,n} = 100 [\log(P_{t,n}) - \log(P_{t,n-1})]$, $t = 1, 2, \dots, T$, $n = 1, 2, \dots, N$. All $N = 288$ intervals during the 24-hour cycle and $T = 261$ weekdays in the sample are used. To reduce the influence of the slow-trading pattern over the weekend, we follow the adjustment process of Andersen and Bollerslev (1996) by excluding returns from Friday 21:00 GMT through Sunday 21:00 GMT. There are 75167 returns²⁵ for five-minute intervals after the adjustment for the weekend periods.

Appendix 2: Data transformation

Consider the following extreme case.

Let $R_{t,n}$ be 12 hours data ($k = 1$). The two first returns are $R_{1,1}$ and $R_{1,2}$ while the corresponding filters are $\hat{s}_{1,1}$ and $\hat{s}_{1,2}$. Following Andersen and Bollerslev (1997b), the first filtered 24 hours (1 day, $k = 2$) return is:

$$\tilde{R}_{1,1}^{(2)} = \frac{R_{1,1}}{\hat{s}_{1,1}} + \frac{R_{1,2}}{\hat{s}_{1,2}} = \frac{R_{1,1}\hat{s}_{1,2} + R_{1,2}\hat{s}_{1,1}}{\hat{s}_{1,1}\hat{s}_{1,2}}$$

Recalling that if we only include in the FFF variables that are related to a one day horizon (excluding for instance daily effects), $\frac{1}{N} \sum_{i=1}^N \hat{s}_{i,t} = 1$, which means that $\tilde{R}_{1,1}^{(2)}$ should be equal to $R_{1,1}^{(2)}$. However, following Andersen and Bollerslev (1997b):

²³Papers by Ito and Roley (1987) and Ederington and Lee (1993) suggest that sampling frequencies as short as one hour may be too long to assess the impact of macroeconomic announcement on volatility accurately. So, we compute paces at five-minute intervals because we study announcements.

²⁴It is an interpolation between the preceding and immediately following quotes weighted linearly by their inverse relative distance to the desired point in time.

²⁵To preserve the number of returns associated with one week we make no corrections for any worldwide or country specific holidays that occurred during the sample period.

$$\tilde{R}_{1,1}^{(2)} = \frac{R_{1,1}\hat{s}_{1,2} + R_{1,2}\hat{s}_{1,1}}{\hat{s}_{1,1}\hat{s}_{1,2}} \neq R_{1,1} + R_{1,2}.$$

Figures:

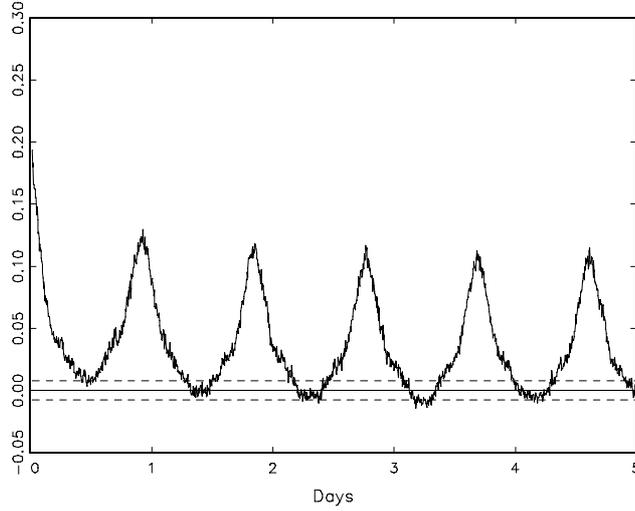


Figure 1: Five days correlogram of five-minute absolute returns.

Note: The figure plots the lag 5 through 1440 sample autocorrelation for the five-minute absolute returns on the DM-USD from October 1, 1992 through September 30, 1993. The 95% Bartlett confidence bands for no serial dependence are also reported in the figure.

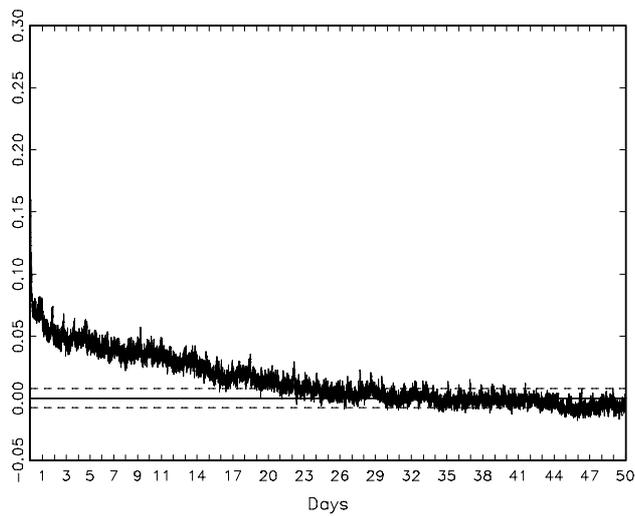


Figure 2: Fifty days correlogram of filtered (intraday periodic components and announcement effects) five-minute absolute returns

Note: The figure plots the lag 5 through 14400 sample autocorrelation for the filtered five-minute absolute returns on the DM-USD from October 1, 1992 through September 30, 1993. The 95% Bartlett confidence bands for no serial dependence are also reported in the figure. We define the filtered five-minute return series as $\tilde{R}_{t,n} \equiv R_{t,n}/\hat{s}_{t,n}$ where $\hat{s}_{t,n}$ is the estimator of the intraday periodic component for interval n on day t . We use a Flexible Fourier form to pre-filter the data for seasonality.

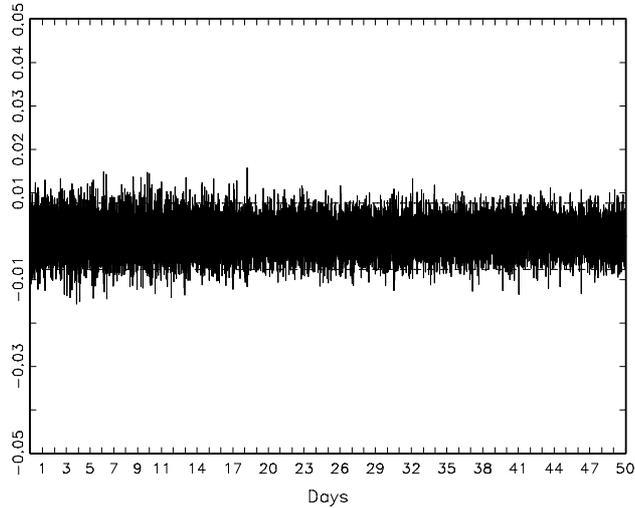


Figure 3: Fifty days correlogram of the fractionnally differenced filtered five-minute absolute returns ($d = 0.04$)

Note: The figure graphs the lag 5 through 14400 sample autocorrelation for the fractionnally differenced filtered five-minute absolute returns, $(1-L)^{0.4} |\tilde{R}_{t,n}|$ where $t=1,2,\dots,261$, $n=1,2,\dots,288$. The 95% Bartlett confidence bands for no serial dependence are also reported in the figure.

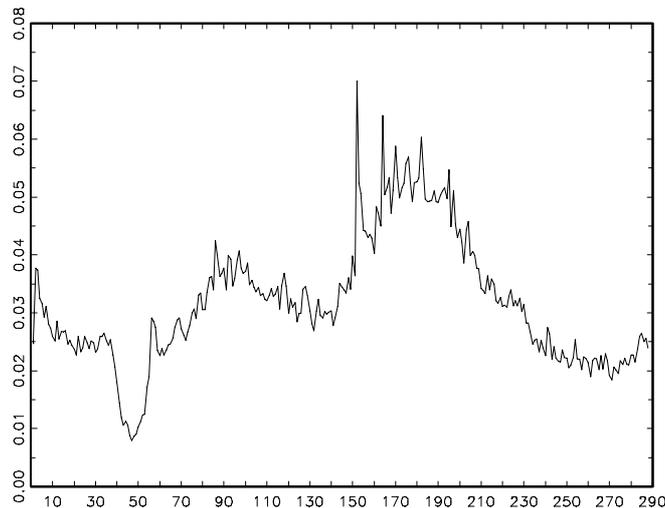


Figure 4: Intraday average absolute returns for the DM-USD

Note: The figure graphs the average absolute five-minute return for each five-minute interval, starting with the interval 0:00-0:05 GMT and ending at 23:55-0:00 GMT. The returns are calculated from a linear interpolation (for more details, see appendix 1) over the October 1, 1992 to September 30, 1993 sample

period. Quotes from Friday 21:00 GMT through Sunday 21:00 GMT are excluded, resulting in a total of 75167 return observations. All 261 weekdays are employed in calculating the averages.

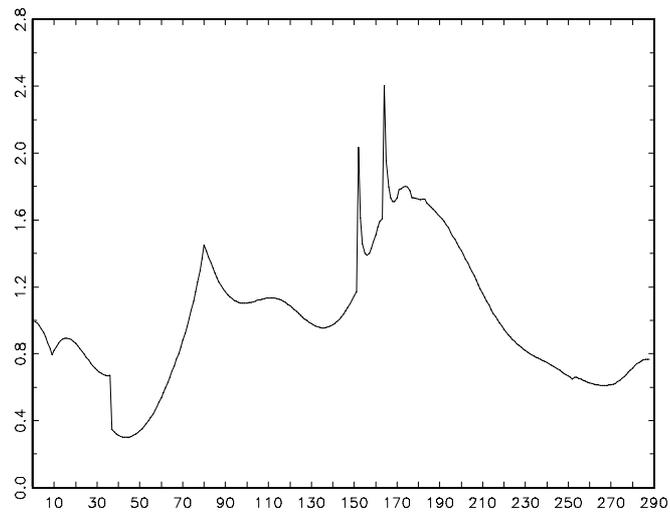


Figure 5: Average Flexible Fourier functional form of intraday five-minute returns for the DM-USD